Completing the Square

The method of completing the square is sometimes used to convert a quadratic function in this form $f(x)=ax^2+bx+c$ into vertex form $f(x)=a(x-h)^2+k$ where the vertex is $(h,k)$.

Start with a quadratic function in $ax^2+bx+c$ form.

Separate the first two terms with parentheses.

Factor out the constant $a$ from each term in the parentheses. Leave room after the $\frac{b}{a}x$ term.

Take the coefficient in front of the $x$ term and divide it by 2 then square it. $\frac{b}{a} \rightarrow \left(\frac{b}{2a}\right)^2$. Add that value inside the parentheses.

The value $\left(\frac{b}{2a}\right)^2$ was added inside the parentheses and everything inside the parentheses is being multiplied by $a$ so you have actually added $a\left(\frac{b}{2a}\right)^2$ to the entire function. For everything to remain equal we must subtract $a\left(\frac{b}{2a}\right)^2$ from the entire function. Subtract $a\left(\frac{b}{2a}\right)^2$ outside the parentheses.

$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2$ is now a perfect square so it can be condensed into $\left(x + \frac{b}{2a}\right)^2$.

$f(x)$ is now in vertex form with the vertex $(h,k)$ being $\left(-\frac{b}{2a}, c - a\left(\frac{b}{2a}\right)^2\right)$. You have now completed the square.
Examples

Start with $f(x) = x^2 + 2x - 17$.  

Separate the first two terms with parentheses.

There is no need to factor out the coefficient in front of the $x^2$ term because it is already 1. Divide the coefficient in front of the $x$ term by 2 then square it. 

$2 \rightarrow \left(\frac{1}{2}\right)^2 = 1$. Add 1 inside the parentheses.

Since you added 1 to the function, you must subtract 1 from the function to keep everything equal. Subtract 1 outside the parentheses.

$x^2 + 2x + 1$ is a perfect square so it can be condensed into $(x + 1)^2$.

$f(x)$ is now in vertex form. The vertex is $(-1, -18)$.

Start with $f(x) = 5x^2 - 3x + 20$.

Separate the first two terms with parentheses.

Factor 5 out of the parentheses.

Divide $-\frac{3}{5}$ by 2 then square it. $-\frac{3}{5} \rightarrow \left(-\frac{3}{10}\right)^2 = \frac{9}{100}$.

Add $\frac{9}{100}$ inside the parentheses.

The $\frac{9}{100}$ is being multiplied by 5 so you must subtract $5\left(\frac{9}{100}\right)$ from the function to keep everything equal.

$5\left(\frac{9}{100}\right) = \frac{9}{20}$. Subtract $\frac{9}{20}$ outside the parentheses.

$x^2 - \frac{3}{5}x + \frac{9}{100}$ is a perfect square which can be condensed into $(x - \frac{3}{10})^2$.

$f(x)$ is now in vertex form. The vertex is $\left(\frac{3}{10}, \frac{391}{20}\right)$. 