Making Something from Nothing: The Banach Tarski Paradox

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The Banach-Tarski Paradox asserts that a solid ball can be decomposed into finitely many pieces and reassembled to form two balls identical to the original. This presentation will provide a systematic exploration of this result, examining the mathematical foundations that make it possible and presenting an approachable outline of the proof. I will discuss the foundational ideas of set theory and geometry required to understand this construction, focusing particularly on the Axiom of Choice. In the end, I discuss the importance of this paradox under the context of Set Theory and mathematics as a whole, highlighting how it challenges our geometric intuitions and illustrates the surprising consequences of foundational mathematical assumptions.